

Atelier “CHIMIE DES PLASMAS” – CINÉTIQUE DES ÉTATS EXCITÉS – MISE EN ŒUVRE DE MODÈLES COLLISIONNELS-RADIATIFS (CR)

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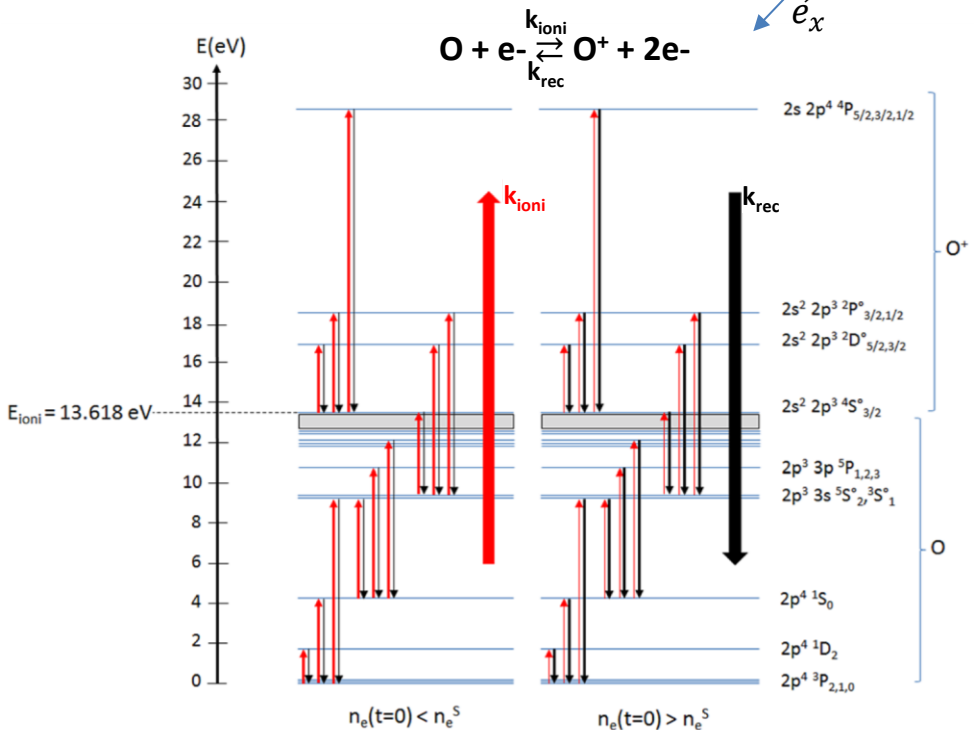
Plan

1. Chimie des états excités **vs** chimie des espèces :
identification de constantes de vitesse globale
2. Bilan détaillé vrai pour les constantes de vitesse globale ?
3. Diagrammes de BOLTZMANN et de SAHA-BOLTZMANN /
Elaboration modèle Collisionnel-Radiatif et illustration

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Kinetics – Excited states and species



$$k_{m \rightarrow n} = \sqrt{\frac{8k_B T_e}{\pi m_e}} \int_{\frac{E_n - E_m}{k_B T_e}}^{+\infty} \sigma_{m \rightarrow n}(x) x e^{-x} dx$$

Individual O_m variation rate

$$\begin{aligned} \frac{1}{V} \frac{dN_{O_m}}{dt} = & - \sum_{n>m} k_{m \rightarrow n} \left(1 - \frac{[O_n]}{[O_m]K_{n,m}^B}\right) [O_m] n_e & k_{m \rightarrow n} \begin{array}{c} n \\ \nearrow \\ m \end{array} \\ & + \sum_{n<m} k_{n \rightarrow m} \left(1 - \frac{[O_m]}{[O_n]K_{m,n}^B}\right) [O_n] n_e & k_{n \rightarrow m} \begin{array}{c} n \\ \nwarrow \\ m \end{array} \\ & - \sum_i k_{m \rightarrow i}^+ \left(1 - \frac{[O_i^+] n_e}{[O_m] K_{i,m}^S}\right) [O_m] n_e & k_{m \rightarrow i}^+ \begin{array}{c} i + e^- \\ \nearrow \\ m \end{array} \end{aligned}$$

E_{ioni}

Individual O_i^+ variation rate

$$\begin{aligned} \frac{1}{V} \frac{dN_{O_i^+}}{dt} = & - \sum_{j>i} k_{i \rightarrow j} \left(1 - \frac{[O_j^+]}{[O_i^+] K_{j,i}^B}\right) [O_i^+] n_e & k_{i \rightarrow j} \begin{array}{c} j \\ \nearrow \\ i \end{array} \\ & + \sum_{j<i} k_{j \rightarrow i} \left(1 - \frac{[O_i^+]}{[O_j^+] K_{i,j}^B}\right) [O_j^+] n_e & k_{j \rightarrow i} \begin{array}{c} i \\ \nwarrow \\ j \end{array} \\ & + \sum_m k_{m \rightarrow i}^+ \left(1 - \frac{[O_i^+] n_e}{[O_m] K_{i,m}^S}\right) [O_m] n_e & k_{m \rightarrow i}^+ \begin{array}{c} i \\ \nearrow \\ m \end{array} \end{aligned}$$

Global O variation rate

$$\frac{1}{V} \frac{dN_O}{dt} = \sum_m \frac{1}{V} \frac{dN_{O_m}}{dt} = \begin{cases} -k_{ioni}[O]n_e \\ +k_{rec}[O^+]n_e^2 \end{cases}$$

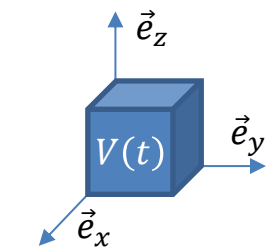
Global O^+ variation rate

$$\frac{1}{V} \frac{dN_{O^+}}{dt} = \sum_i \frac{1}{V} \frac{dN_{O_i^+}}{dt} = \begin{cases} +k_{ioni}[O]n_e \\ -k_{rec}[O^+]n_e^2 \end{cases}$$

J. Annaloro, V. Morel, A. Bultel *et al.*
[Phys. Plasmas **19** \(2012\) 073515](#)

Kinetics of ionization – Example

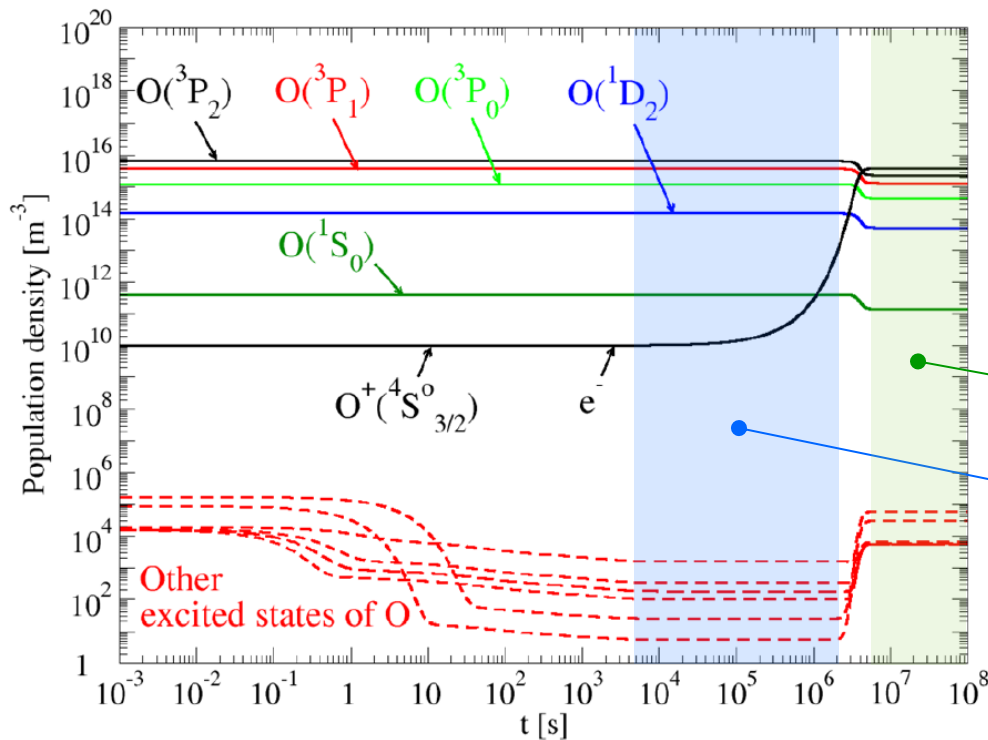
0D simulation



For ionization:

$$n_e(t=0) < n_e^S$$

$$= K^S \left(\sqrt{1 + \frac{p}{K^S k_B T_e}} - 1 \right)$$



$$T_e = 6000 \text{ K}$$

$$n_e(t=0) = 10^{10} \text{ m}^{-3} \quad p = 10^{-3} \text{ Pa}$$

$$T_{exc}(t=0) = 6000 \text{ K}$$

Chemical equilibrium
Excitation equilibrium

Quasi Steady State QSS

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[Phys. Plasmas **19** \(2012\) 073515](#)

Identification of global rate coefficient

Global O variation rate

$$\frac{1}{V} \frac{dN_O}{dt} = \sum_m \frac{1}{V} \frac{dN_{Om}}{dt} = \begin{cases} -k_{ioni}[O]n_e \\ +k_{rec}[O^+]n_e^2 \end{cases}$$

Global O⁺ variation rate

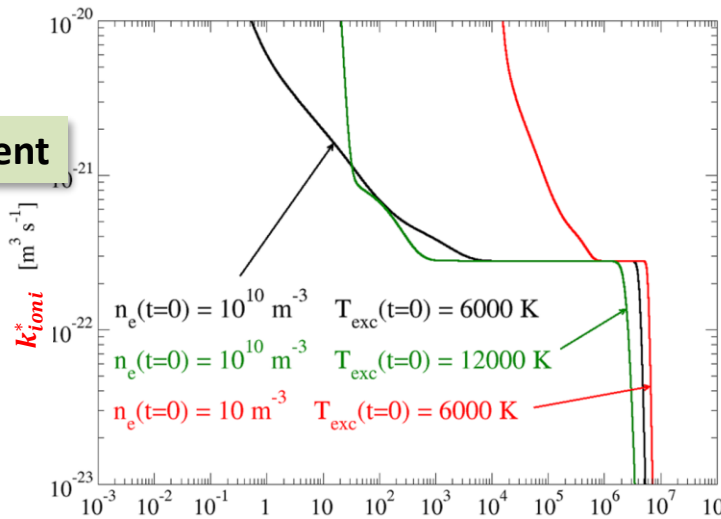
$$\frac{1}{V} \frac{dN_{O^+}}{dt} = \sum_i \frac{1}{V} \frac{dN_{O_i^+}}{dt} = \begin{cases} +k_{ioni}[O]n_e \\ -k_{rec}[O^+]n_e^2 \end{cases}$$

For ionization, we follow:

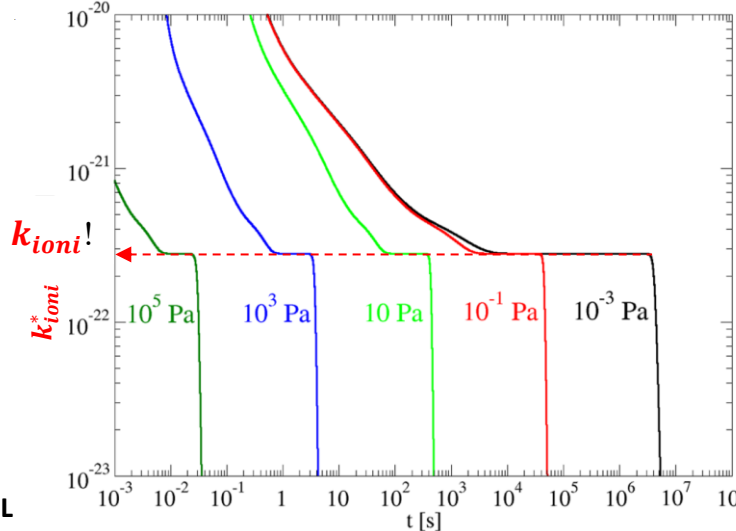
$$k_{ioni}^* = - \frac{1}{V[O]n_e} \frac{dN_O}{dt} = + \frac{1}{V[O]n_e} \frac{dN_{O^+}}{dt}$$

For recombination, we follow:

$$k_{rec}^* = + \frac{1}{V[O^+]n_e^2} \frac{dN_O}{dt} = - \frac{1}{V[O^+]n_e^2} \frac{dN_{O^+}}{dt}$$

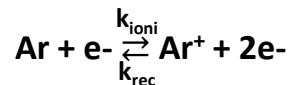


$T_e = 6000 \text{ K}$
 $n_e(t=0) = ??? \text{ m}^{-3}$ $p = 10^{-3} \text{ Pa}$
 $T_{exc}(t=0) = ??? \text{ K}$



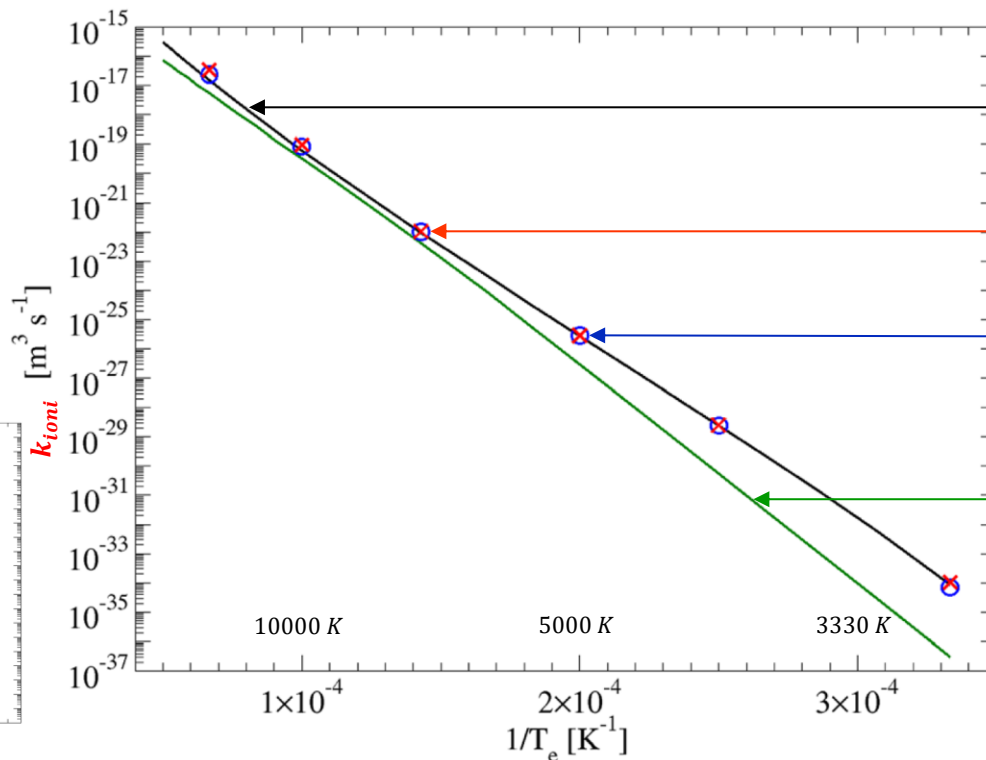
$T_e = 6000 \text{ K}$
 $n_e(t=0) = 10^{10} \text{ m}^{-3}$ $p = ??? \text{ Pa}$
 $T_{exc}(t=0) = 6000 \text{ K}$

Influence of temperature and energy diagram



Arrhenius-type rate coefficient

$$k_{\text{ioni}}(T_e) = A T_e^\alpha e^{-\frac{E_A}{k_B T_e}}$$



NIST database (previous approach)

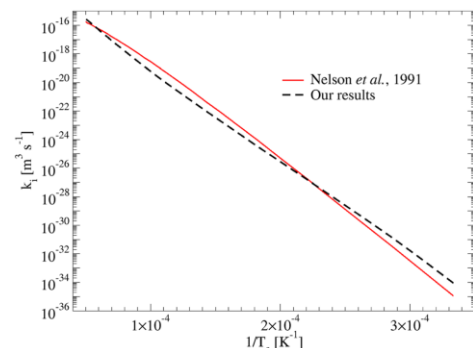
NIST database
+ Rydberg states with $g_k = 1$

NIST database
+ Rydberg states with $g_k = 2n_k^2$

Vleck energy diagram

J. Vlcek, *J. Phys. D: Appl. Phys.* **22** (1989) 623

J. Annaloro, V. Morel, A. Bultel *et al.*
Phys. Plasmas **19** (2012) 073515



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Detailed balance

Individual O_m variation rate

$$\frac{1}{V} \frac{dN_{O_m}}{dt} = - \sum_{n>m} k_{m \rightarrow n} \left(1 - \frac{[O_m]}{[O_m] K_{n,m}^B} \right) [O_m] n_e \quad \begin{array}{c} n \\ \xrightarrow{k_{m \rightarrow n}} m \end{array}$$

$$+ \sum_{n<m} k_{n \rightarrow m} \left(1 - \frac{[O_m]}{[O_n] K_{m,n}^B} \right) [O_n] n_e \quad \begin{array}{c} m \\ \xrightarrow{k_{n \rightarrow m}} n \end{array}$$

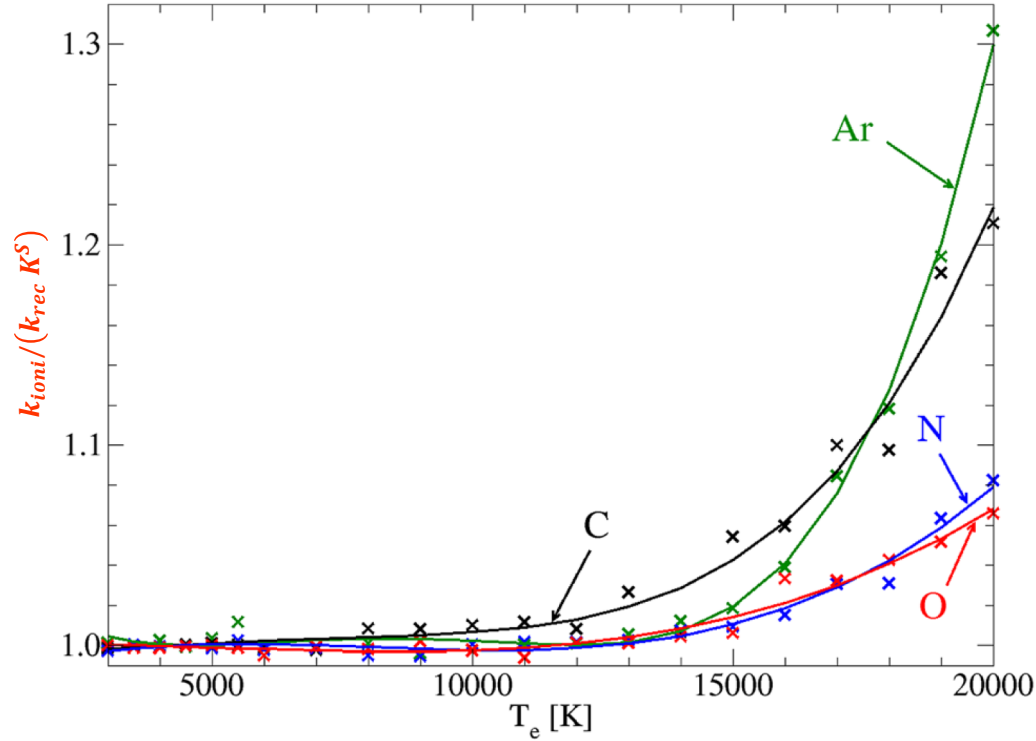
$$- \sum_i k_{m \rightarrow i}^+ \left(1 - \frac{[O_i^+] n_e}{[O_m] K_{i,m}^S} \right) [O_m] n_e \quad \begin{array}{c} i + e^- \\ \xrightarrow{k_{m \rightarrow i}^+} m \end{array} \quad E_{ioni}$$

BOLTZMANN equilibrium constant $K_{n,m}^B = \frac{g_n}{g_m} e^{-\frac{E_n - E_m}{k_B T_e}}$

SAHA equilibrium constant $K_{i,m}^S = \frac{2 g_i^+}{g_m} \left(\frac{2\pi m_e k_B T_e}{h^2} \right)^{3/2} e^{-\frac{E_i - E_m}{k_B T_e}}$

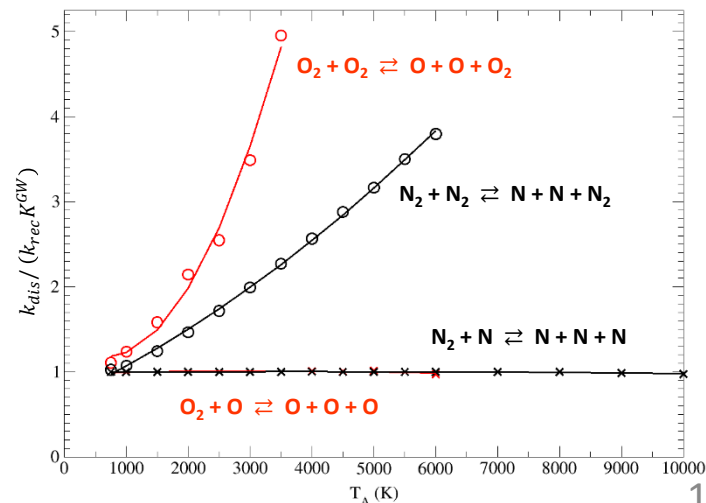
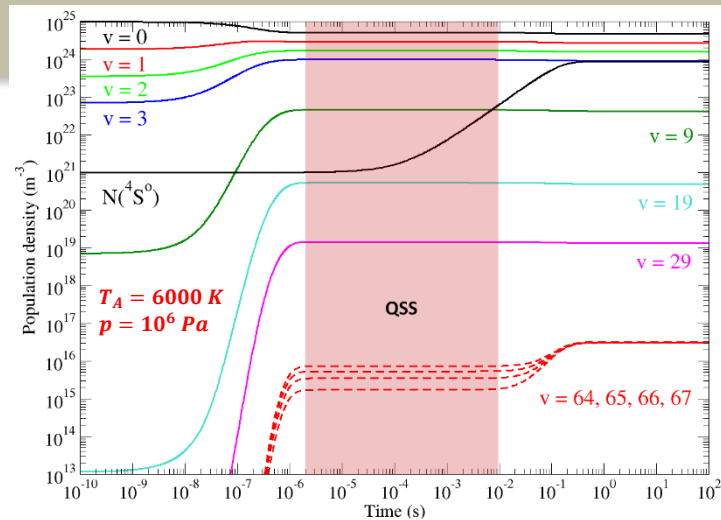
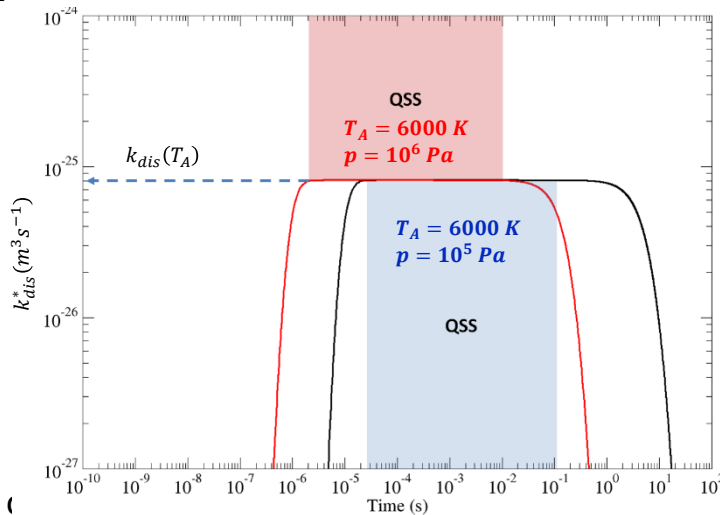
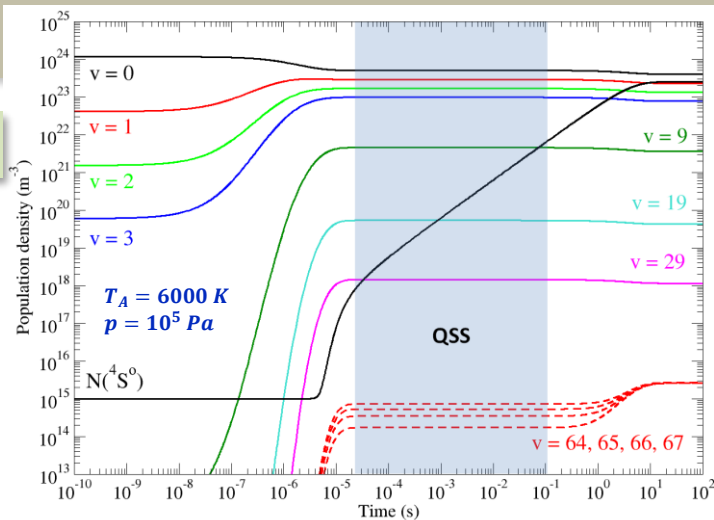
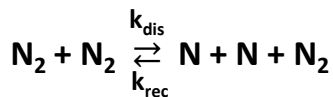
Species SAHA equilibrium constant $K^S = \frac{2 Z^+}{Z} \left(\frac{2\pi m_e k_B T_e}{h^2} \right)^{3/2} e^{-\frac{E_{ioni}}{k_B T_e}}$

$$O + e^- \xrightleftharpoons[k_{rec}]{k_{ioni}} O^+ + 2e^- \quad \frac{1}{V} \frac{dN_{O^+}}{dt} = k_{ioni}[O]n_e - k_{rec}[O^+]n_e^2 \xrightarrow{\text{Chemical eq.}} K^S = \frac{n_e[O^+]}{[O]} = \frac{k_{ioni}}{k_{rec}} \Rightarrow \frac{k_{ioni}}{k_{rec} K^S} = 1$$



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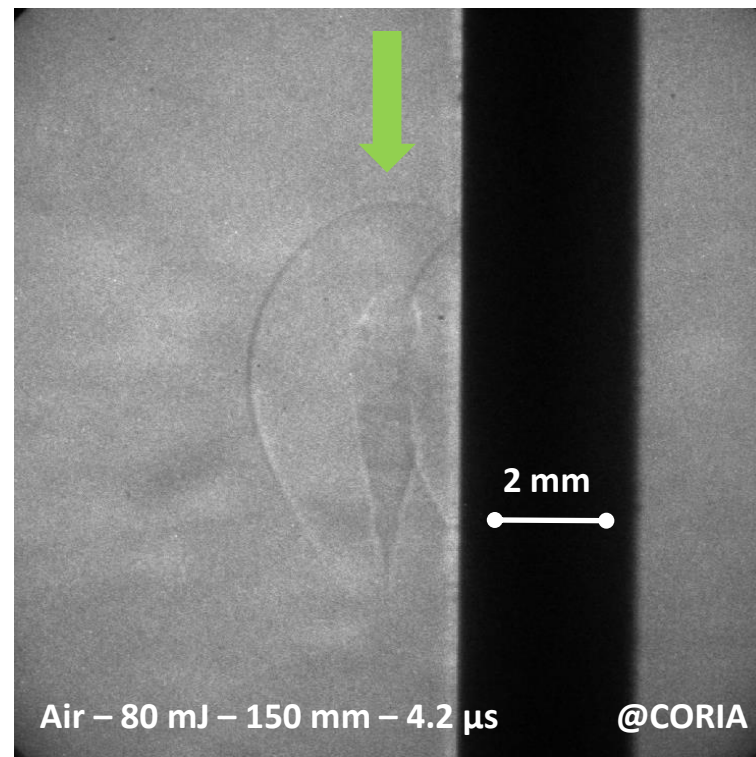
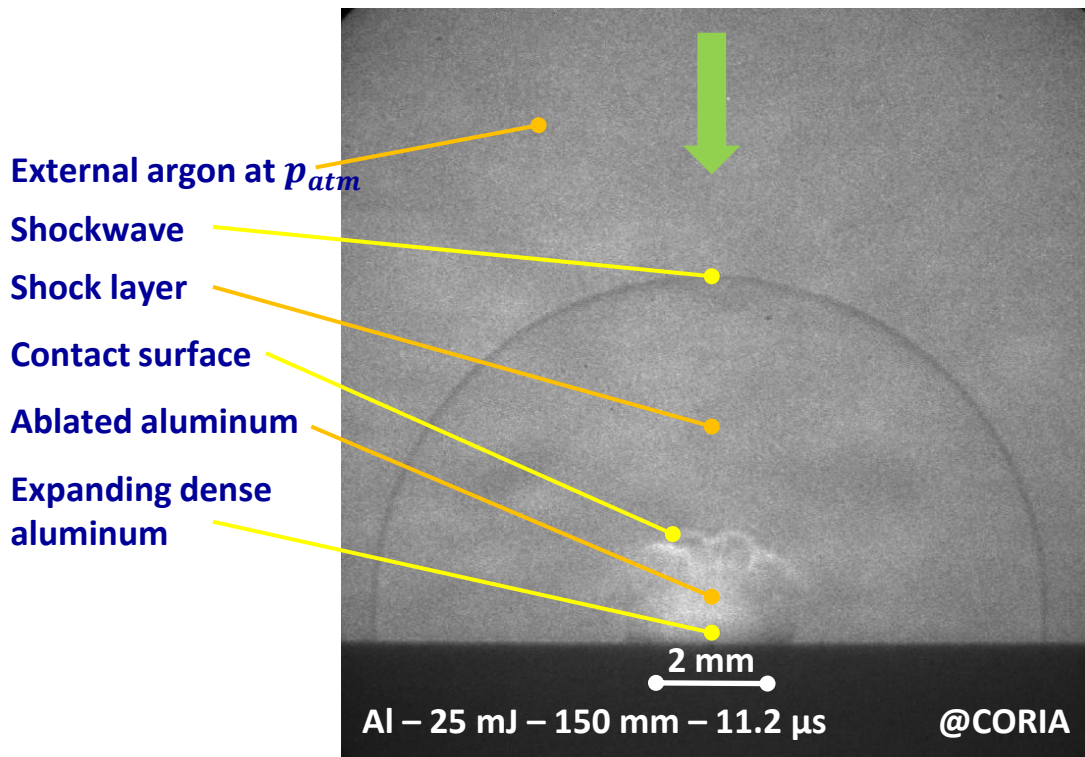
Detailed balance



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Influence of radiation?



The ECHREM* code

*Eulerian Chemically REactive
Multi-component plasma code



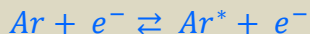
V. Morel, A. Bultel *et al.*

[Spectrochim. Acta B 127 \(2017\) 7-19](#)

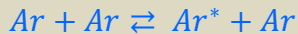
Shock layer - Argon

Collisional-Radiative model CoRaM-RG

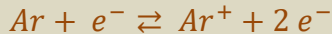
Ar (5 lev.)
Ar⁺ (1 lev.)
Ar₂⁺ (1 lev.)



Exc. Elec. Impact



Exc. Elec. Impact



Ioni. Elec. Impact



Ioni. Elec. Impact



Ioni. Heavy Impact



Ioni. Heavy Impact



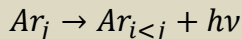
Penning Ioni.



Disso. Recomb.



Rad. Recomb.



Spont. Emiss.

500 elementary processes

Collisional Database

$$k_i(T_{A,e}) = \sqrt{\frac{8 k_B T_{A,e}}{\pi \mu}} \int_{x_0}^{+\infty} x e^{-x} \sigma_i(x) dx \text{ with}$$

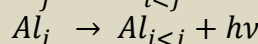
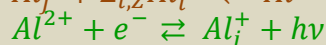
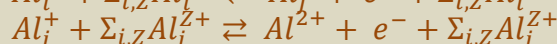
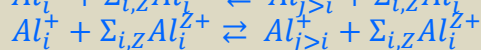
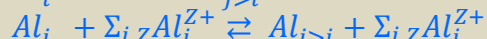
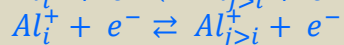
- $\sigma_i(x)$ collisional cross section and
- $x = \frac{\varepsilon}{k_B T_{A,e}}$ reduced collision energy

Backward rate coefficient deduced from the **Detailed Balance**

Central plasma - Aluminum

Collisional-Radiative model CoRaM-Al

Al (80 lev.)
Al⁺ (170 lev.)
Al²⁺ (1 lev.)



Thermal Bremsstrahlung

65 000 elementary processes

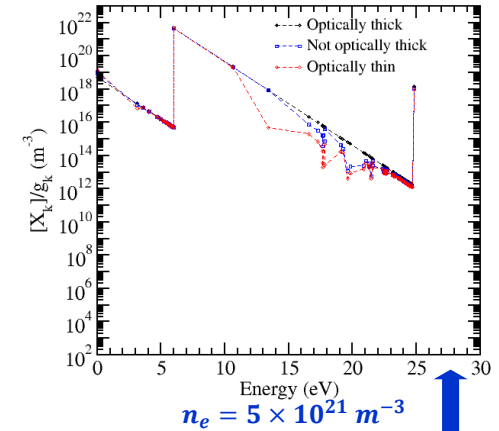
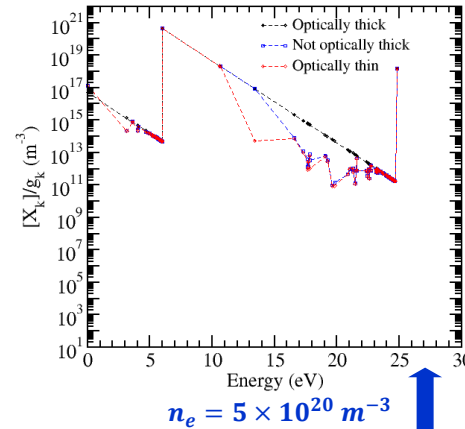
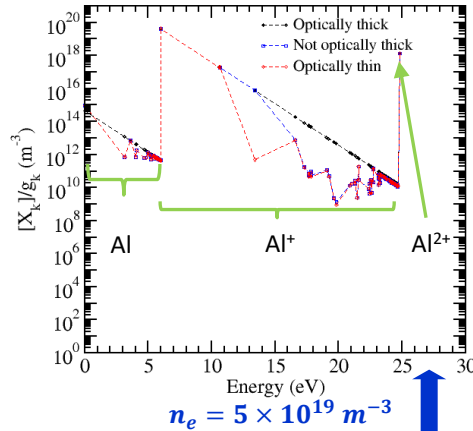
Radiative Database

NIST, Atomic Line List, ADAS, HULLAC...

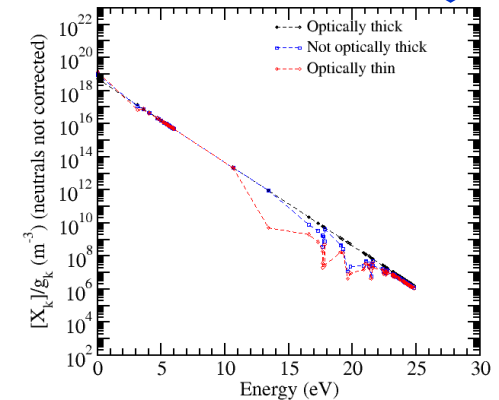
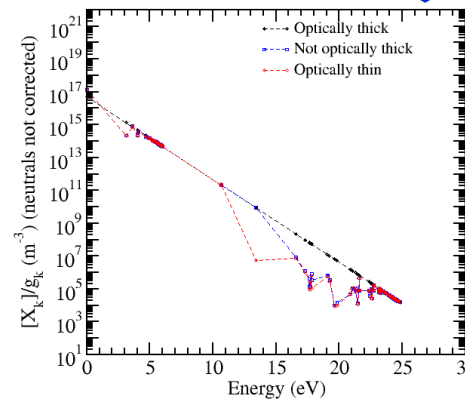
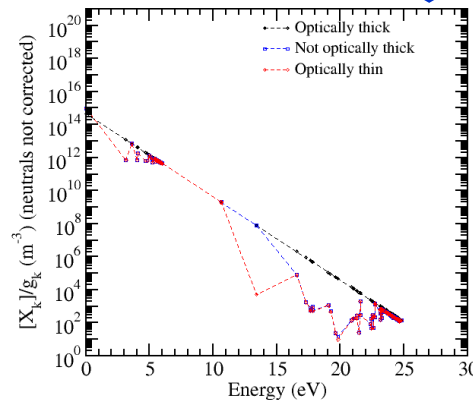
BOLTZMANN and SAHA-BOLTZMANN plots

- Recombination under almost constant n_e
- Constant temperature $T_e = 10\,000\text{ K}$ (order of magnitude for laser-induced Aluminum plasma)
- Constant radius $R = 1\text{ mm}$ (order of magnitude for laser-induced Aluminum plasma)
- BOLTZMANN and SAHA-BOLTZMANN plots at $t = 1\text{ }\mu\text{s}$

BOLTZMANN plots



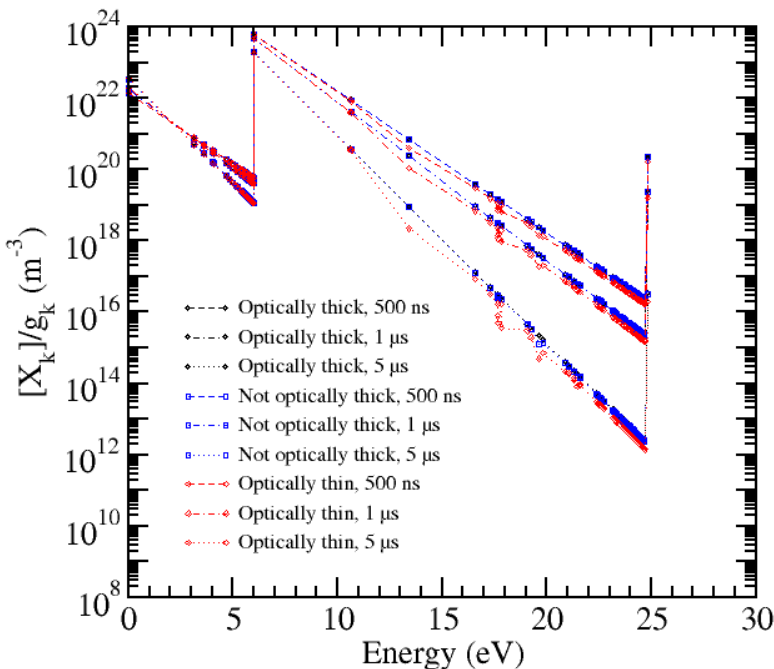
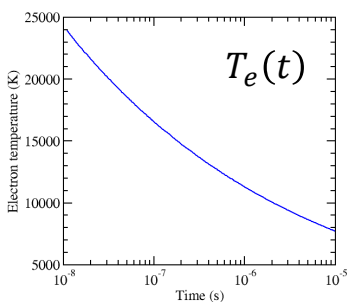
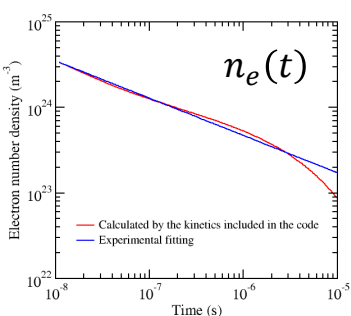
SAHA-BOLTZMANN plots



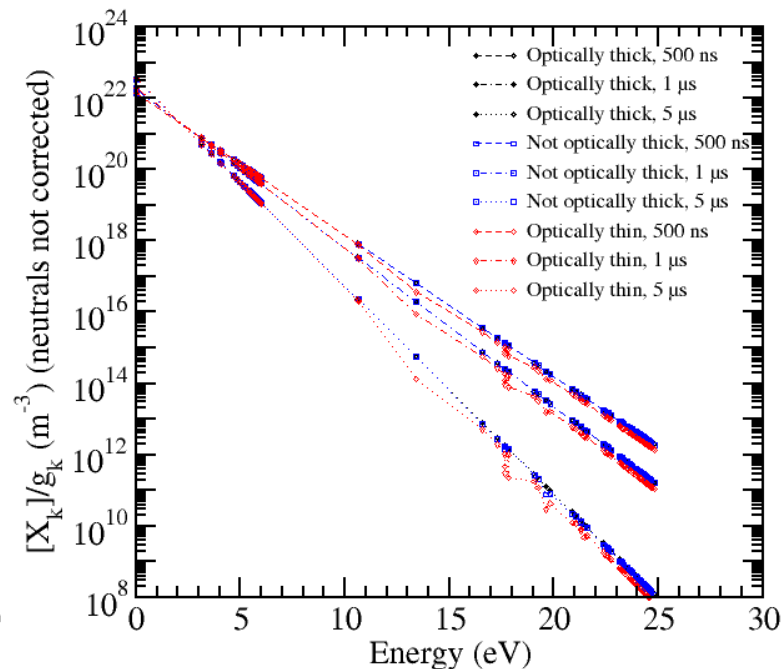
BOLTZMANN and SAHA-BOLTZMANN plots

EXPERIMENTAL TIME-EVOLUTION (typical conditions for a laser-induced Aluminum plasma)

- Electron density $n_e(t) = 3.5 \times 10^{24} \left(\frac{t[\text{ns}]}{10}\right)^{-0.436} \text{ m}^{-3}$ for $t > 10 \text{ ns}$
- Electron temperature $T_e(t) = 24\,270 \left(\frac{t[\text{ns}]}{10}\right)^{-0.166} \text{ K}$ for $t > 10 \text{ ns}$
- Plasma radius $R(t) = 0.63 \left(\frac{t[\text{ns}]}{10}\right)^{0.1} \text{ mm}$ for $t > 10 \text{ ns}$



BOLTZMANN plot



SAHA-BOLTZMANN plot